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INCORPORATION OF NON-DESTRUCTIVE CENTRIFUGE TESTS INTO MISSILE GUIDANCE  
ASSESSMENT

Dean A. Payne  
The Johns Hopkins University Applied Physics Laboratory  
Laurel, MD

Harvey Feltquate  
Charles Stark Draper Laboratory  
Cambridge, MA

Abstract

As an adjunct to the Navy's surveillance program for guidance assessment, the FBM Guidance Branch is considering a test program in which operational Trident II guidance system fleet assets would be "flown" on a centrifuge. This paper outlines an approach to the analysis of the guidance data. A test for tactical representativeness is described, as well as an approach to the optimal combination of centrifuge data with test flight data, provided that the test for tactical representativeness is passed. The approach relies on a mathematical error model structure that relates fundamental level errors (accelerometer and gyro biases, etc.) to instrumented measurements of position and/or velocity along the guidance system's trajectory. With multiple tests this structure allows maximum likelihood estimation techniques to be applied to estimate the mean and variance of the fundamental guidance errors. The inverse of the Fisher information matrix associated with the estimate defines the uncertainty in the estimate and allows a quantitative test of the centrifuge-based estimate against a similar flight-based estimate. If the test is passed, the information-weighted average of the two estimates yields the optimal estimate.

Background

Estimates of the reliability and accuracy of the Trident II guidance system have always been based on flight test data to the exclusion of factory or lab tests, arguing that flight tests are tactically representative in virtually every respect as far as guidance is concerned, while factory and lab tests may not be. As the number of operational weapon system flight tests flown each year has diminished the Navy is

considering modifications of the surveillance program in which additional non-destructive tests would be used to improve early detection of degradation in the performance, reliability, and functionality of the system from year to year, so that remedial action can be taken in a timely manner. Early detection with sufficient lead time is required to repair or replace defective components to prevent systematic degradation. Upon completion of testing, these assets will be returned to the fleet. For the assessment of changes in performance, a prime candidate is a test in which an operating Trident II guidance inertial measurement unit (IMU) and electronics assembly (EA) would be "flown" in a controlled environment, e.g., on a centrifuge. This paper presents a methodology by which the centrifuge data can be tested for tactical representativeness (comparing to flight test data) and, if deemed representative, combined with data from the relatively few test flights per year to provide an optimal estimate of guidance accuracy. While reliability and functionality will be measured and tracked with a centrifuge, this paper specifically does not address these aspects of the surveillance program.

Typically, flight test data is used in a "shoot and score" approach to estimate the accuracy of a system. This is a straightforward approach that requires little processing to provide the desired estimate — usually a computation of the sample mean and variance of impact misses. If the Navy had taken this approach to flight-based evaluation of Trident II accuracy, it would be nearly impossible to compare or incorporate centrifuge data with flight test data in any quantifiable way. The centrifuge data does not, and cannot, provide any "impact miss" to compare with the impact misses of the flight tests. Fortunately, however,

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the Navy has adopted a much more sophisticated approach to the estimation of system accuracy, based, not on impact miss data alone, but on data from a fully instrumented system that allows one to estimate the statistical parameters of the fundamental error sources (e.g., accelerometer and gyro biases, etc.) that cause the inaccuracy. It is this same approach, which, when applied to centrifuge data, allows the quantifiable statistical comparison of centrifuge and flight data and permits the optimal combining of centrifuge and flight data. The next section provides an overview of the methodology the Navy uses on Trident II test flights and shows how it can be applied equally well to the processing of centrifuge test data.

### Methodology

Given that a Trident II missile is to be flight-tested, it would seem remiss not to take the opportunity to gather much more information than the simple impact miss, by providing instrumentation yielding insight into how the different subsystems perform during the various phases of flight. This additional data, when properly processed, will provide much better estimates of missile performance than the "shoot and score" approach, both in terms of understanding, and in terms of the tightness of the confidence bounds associated with the estimate. From the inception of the Trident II flight test program the Navy has planned for this enhanced data gathering and processing so that the instrumentation is an integral part of the tactical system design. Thus, the instrumentation does not affect the tactical representativeness of the test and all instrumented data is representative of a tactical system.

For assessment of guidance accuracy the instrumentation is a GPS-based missile tracking system called SATRACK II that provides precise measurements of position and velocity along the trajectory. Missile telemetry provides the guidance system's estimate of position and velocity which is differenced with the SATRACK estimate to provide an estimate of the guidance system error. (In actuality, it is the difference in range and range rate to the various satellites that are the measured quantities.) Multiple measurements of the error in the guidance system's computation of position and

velocity do not readily translate into estimates of the bias in an accelerometer or gyro, so how does the SATRACK data enable one to estimate fundamental level errors (e.g., accelerometer and gyro biases, etc.)?

The key to the Navy's approach to Trident II accuracy evaluation lies in the reliance on a weapon system error model structure that relates the observed errors at various times along the trajectory to the fundamental level errors that are assumed to be the cause of the position and velocity errors. This error model structure takes the form of a first order linear system of differential equations obtained by first order perturbation of the guidance navigation equations and by basic physics which describes how the various fundamental level errors affect acceleration and platform orientation. In fact, letting  $x(t)$  be the 9-vector of position, velocity, and platform orientation errors at time  $t$  relative to some inertial reference frame, and  $y(t)$  be the vector of accelerometer, gyro, and clock errors at time  $t$ , the model structure takes the form of a first order matrix differential equation

$$\begin{aligned}\dot{x}(t) &= F(t)x(t) + G(t)y(t) \\ \dot{y}(t) &= 0\end{aligned}\tag{1}$$

with initial conditions given by

$$\begin{aligned}x(t_0) &= x_0 = \text{initial condition errors,} \\ y(t_0) &= y_0 = \text{accelerometer, gyro, and clock errors.}\end{aligned}$$

The accelerometer, gyro, and clock errors  $y$  can be modeled as constants ( $\dot{y}(t) = 0$ ) since the Trident II Mk-6 guidance system inertial components are very stable and the boost phase time of flight is relatively short.  $F(t)$  and  $G(t)$  are functions of time only through their dependence on the specific force (or thrust) vector and on the missile's position (necessary for the evaluation of gravity and its gradient) at time  $t$ , both of which are very much trajectory dependent. The differential equation can therefore be solved if we have a 3-degree-of-freedom missile trajectory simulation to provide a specific force and position history, or if we have telemetry from an actual test flight to provide the same quantities. (If the guidance system were a strapdown system, a 6-degree-of-freedom simulator would be necessary to provide the orientation and angular rates that are not

required for an inertially stable platform.) The solution  $x(t)$  will clearly be a function of the trajectory as well as of the initial errors  $x_0, y_0$ . It can therefore be written as  $x(t; x_0, y_0)$ . The partial derivative of this function with respect to  $x_0$  yields the sensitivity coefficient of the position, velocity, and orientation at time  $t$  to the initial condition position, velocity, and orientation errors (usually called, simply, the *initial condition errors*). Similarly the partial derivative of this function with respect to  $y_0$  (equal to  $y$  since  $\dot{y} = 0$ ) yields the sensitivity coefficient of the position, velocity, and orientation error at time  $t$  to the fundamental level guidance errors  $y$ . In particular, when  $t$  is the time of impact, the matrix of partial derivatives with respect to  $x_0, y_0$  (or *partials* for short) yields the sensitivity of impact miss to the initial condition and fundamental level guidance errors. These partials can be computed for any trajectory for which we have either telemetry or a 3-degree-of-freedom simulation, in other words for any trajectory within the operational envelope of the missile, whether or not such a trajectory has ever been flown in the test program.

#### Estimation of Errors on A Single Test

Since these partials matrices can be computed for any time  $t$  along the trajectory, the SATRACK determined position and velocity error at time  $t$  becomes a measurement of a linear combination (determined by the partials matrix at time  $t$ ) of initial condition errors  $x_0$  and fundamental level guidance errors  $y$ . Given a sufficient number of these measurements and sufficient variation in the partials, the system of linear equations

$$Z = H \begin{bmatrix} x_0 \\ y \end{bmatrix} + v, \quad v \sim N(0, R),$$

$$Z = \begin{bmatrix} z_1 \\ \vdots \\ z_N \end{bmatrix}, \quad H = \begin{bmatrix} h_1 \\ \vdots \\ h_N \end{bmatrix}, \quad v = \begin{bmatrix} v_1 \\ \vdots \\ v_N \end{bmatrix},$$

can be solved for  $x_0, y$ . Here  $z_i$  is the SATRACK measurement at time  $t_i$ ,  $h_i$  is the

partial matrix at time  $t_i$ ,  $v_i$  is the measurement noise at  $t_i$  ( $i=1, \dots, N$ ), and  $R$  is the variance of the large vector  $v$  formed by stacking up the individual noise vectors  $v_i$ . Note that  $v \sim N(0, R)$  is meant to indicate that  $v$  is a normal (Gaussian) random vector with mean zero and variance  $R$ . How well the system can be solved for  $x_0, y$  (the *observability* of  $x_0, y$ ) is clearly a function of the trajectory. For example, if a trajectory is flown in such a way that a certain accelerometer feels little or no specific force, then the scale factor error of that accelerometer will not be observable -- the partial with respect to that accelerometer scale factor error will be virtually zero for all  $t$ . The Fisher information on  $x_0, y$  is the matrix

$$I = H'R^{-1}H$$

and it determines the observability of  $x_0, y$  in that its inverse provides the variance of the error in the maximum likelihood estimate of  $x_0, y$  from the data equation

$$Z = H \begin{bmatrix} x_0 \\ y \end{bmatrix} + v, \quad v \sim N(0, R).$$

Note that the Fisher information matrix may not be invertible (if  $x_0, y$  is not fully observable), in which case a generalized inverse is used and is only applicable as a variance matrix when multiplied left and right by a linear combination of errors that is observable (see sections 4a.4 and 4a.7 of Rao<sup>1</sup>). For example, if, for some matrix  $L$ , the linear combination

$$L \begin{bmatrix} x_0 \\ y \end{bmatrix} \text{ is observable, then}$$

$$L I^{-1} L' = L(H'R^{-1}H)^{-1}L'$$

is the variance of the maximum likelihood

estimate of  $L \begin{bmatrix} x_0 \\ y \end{bmatrix}$ , even though the Fisher information is not invertible. Here

$I^{-1} = (H'R^{-1}H)^{-}$  denotes an arbitrary generalized inverse of  $I$ .

In practice the Fisher information for the estimate of the initial condition and guidance errors can be obtained from a Kalman filter/smoothen run with this error model and these SATRACK measurements. It is *not* the usual Bayesian final variance of the estimation error produced by a filter/smoothen, but can be computed from that variance and the prior variance. The Fisher information is independent of any prior information used to initiate the filter/smoothen while the filter/smoothen's final variance is not. Similarly the maximum likelihood estimate of  $x_0, y$  (which can also be computed from filter/smoothen outputs) is independent of the filter/smoothen prior while the filter/smoothen estimate is not.

#### Cumulative Estimation

All of the above estimation theory applies to the estimation of initial condition and guidance errors realized on a single flight. However, given multiple flights, each with its data equation as above, one can apply a similar maximum likelihood estimation procedure to estimate, not errors themselves, but rather the mean and variance that characterize the errors across all the test flights in the sample. In this *cumulative* case the data fed to the estimation algorithm is mathematically equivalent to the stacked set of data equations from each test in the sample and the errors  $x_0, y$  are now assumed to be samples from a normal distribution with unknown mean  $\mu$  and variance  $\Sigma$  to be estimated. Again, associated with the maximum likelihood estimate of the mean vector and variance matrix of the initial condition and guidance errors  $x_0, y$ , is the Fisher information matrix for the mean and variance of  $x_0, y$  (rather than for the errors  $x_0, y$  themselves as in the previous subsection).

If the variance is known and only the mean  $\mu$  is estimated then the error in the maximum likelihood estimate of the mean is distributed

$N(0, I_{\mu}^{-1})$  where  $I_{\mu}$  is the Fisher information on  $\mu$ . Thus, the Fisher information completely characterizes the distribution of the error in the estimate just as it does in the per-test case.

However, when the mean and variance are both estimated, the error in the maximum likelihood estimate of the mean  $\mu$  and variance  $\Sigma$  is no longer Gaussian as it was for the estimate of  $x_0, y$ , or of  $\mu$  alone, so that a mean and variance no longer completely characterize the distribution. However, it can be shown that asymptotically the Fisher information serves the same role for cumulative estimation of the mean and variance as it does for the per-test estimation of  $x_0, y$ . That is, asymptotically, the maximum likelihood estimation error is distributed normally with zero mean, and variance equal to the inverse of the Fisher information matrix (see Chapter 5 [especially pages 363-370] of Rao<sup>1</sup> and Theorem 2 of Hoadley<sup>2</sup>). The natural question that arises at this point is, "How large does the sample have to be before the asymptotic distribution is reasonably accurate?" This will be addressed in the next subsection after it has been shown how to propagate estimates and uncertainties of fundamental level errors into the impact domain to obtain an estimate of weapon system accuracy (e.g., CEP, Circle of Equal Probability).

#### Propagation of Fundamental Level Errors and Uncertainties into The Impact Domain

The primary reason for doing cumulative performance estimation is to provide an estimate of the weapon system accuracy, and hopefully, also, a quantified uncertainty in that estimate (an estimate with no confidence bounds is not very useful). The cumulative estimation described above does not directly provide an estimate of the mean and variance of impact miss as the "shoot and score" approach does. However, what this approach can do that "shoot and score" cannot do is to provide an estimate of accuracy for arbitrary trajectories including *untested* trajectories. "Shoot and score" can only provide estimates of accuracy for the trajectories within the test sample. The cumulative estimation described above relies heavily on the model structure to estimate the mean and variance of the errors in the structure. In the same way it relies on the model structure to generate estimates of the mean and variance of impact misses (and hence weapon system accuracy) for any given trajectory.

As has been shown earlier, given either appropriate telemetry from an actual test flight or a 3-degree-of-freedom simulation for an untested trajectory, one can use the model structure of the differential equation (1) to compute partial matrices, or sensitivity coefficients, of the position and velocity error at any time along the trajectory to the initial condition and fundamental level guidance errors. In particular a partials matrix to impact can be computed, call it  $\Delta$ . Then multiplying this matrix times the estimated mean vector  $\hat{\mu}$  yields the impact mean  $\Delta\hat{\mu}$  for this trajectory. Similarly the variance of impact errors is obtained from the estimated variance matrix  $\hat{\Sigma}$  as  $\Delta\hat{\Sigma}\Delta'$ . The variance of the error in the estimates  $\Delta\hat{\mu}, \Delta\hat{\Sigma}\Delta'$  can be obtained by propagating the inverse of the Fisher information on  $\mu$  and  $\Sigma$  into the impact domain in a similar way, the inverse of the Fisher on the mean being propagated as  $\Delta I_{\mu}^{-1} \Delta'$ , but the

Fisher on the variance requiring some additional computation (involving the tensor product of the partials matrix) to take it to the impact domain. Ultimately, modest computations allow one to predict weapon system accuracy for an arbitrary trajectory and, also, allow one to quantify the uncertainty in the prediction by propagating the inverse of the Fisher information into the impact domain.

At this point we return to the question raised at the end of the last subsection as to when the asymptotic normal distribution of the estimation errors begins to be reasonably accurate, i.e., how large does the sample have to be for the asymptotic distribution to produce reasonable confidence bounds? It should be evident from the earlier discussion of the Fisher information and observability that the sample size alone is not sufficient to answer the question. Observability is equally important. If, in  $N$  tests none of the trajectories excites a particular error, then there can be no inference as to the mean and variance of that error, no matter how large  $N$  may be. A more likely situation is that only one or two tests among the total of  $N$  excite the particular error so that, although the sample size of the program is  $N$ , the effective sample size for the particular error is only one or two yielding poor estimation

capability for its mean and variance. Fortunately, all this is captured in the Fisher information matrix for  $\mu$  and  $\Sigma$  just as it was for the per-test estimation of  $x_0, y$ .

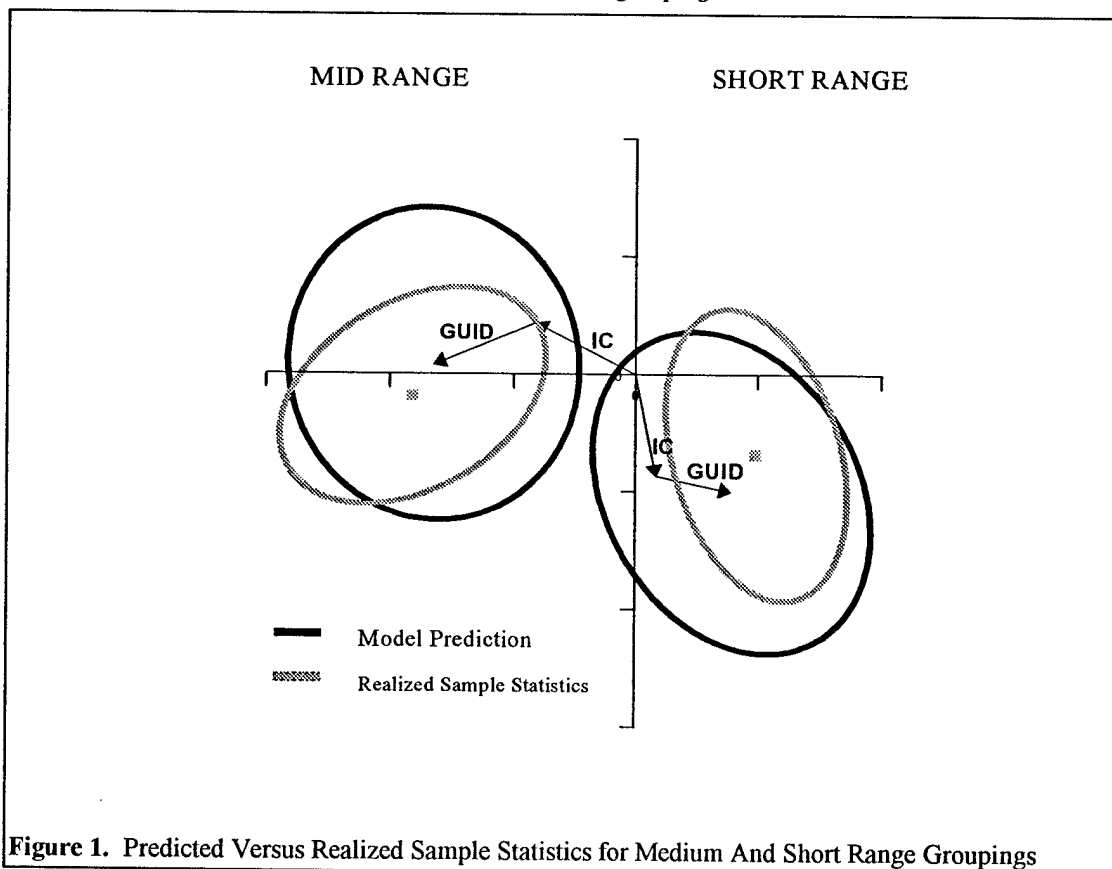
Thus it is likely that for specific error states or linear combinations of error states in the model structure the estimation error will be large. However, our real interest lies in the knowledge of how accurate the weapon system is, i.e., in how all the fundamental level errors manifest themselves at impact. But those fundamental level errors (or, more precisely, linear combinations of fundamental level errors) that have a significant impact effect must also have a significant impact sensitivity associated with them and therefore are likely to be those that are readily observable. The linear combinations of states whose mean and variance are poorly observable are likely to be precisely those combinations of states that are of little interest and have little effect on system accuracy. As a result, one might expect that, although the mean and variance of individual error states at the fundamental level might not be very estimable, the mean and variance propagated to impact might be quite estimable (the information on a large number of states compressed to only two states, downrange and crossrange miss).

Indeed, experience with the Trident II test program and with Monte Carlo simulations has shown that, for the Trident II error model, as few as ten or twelve tests are sufficient for the asymptotically normal distribution to yield reasonably accurate 90% confidence bounds for accuracy at impact, provided the trajectory along which the errors are propagated to impact does not differ dramatically from those in the test sample. However, the beauty of the Fisher information is that, even when propagating the estimates on a trajectory so different from those in the test sample that it excites linear combinations of errors never excited before (and hence not observable in the test program), the propagated inverse Fisher information matrices will alert the analyst by providing very large uncertainties.

This has actually happened in the Trident II program. After twelve tests of medium to long range trajectories, an estimate of the mean

and variance of initial condition and guidance errors was formed that was able to provide good predictions for any trajectory similar to those in the test sample of twelve, and even for typical short range flights. However, it was desired to predict the performance for an upcoming test flight that was to be both short and very low loft. When this trajectory was used to propagate the fundamental level means, variances and uncertainties to the impact domain, the resulting uncertainty was huge. It turned out that for very low loft, short range trajectories, the severe energy wasting that is required excited the *difference* of two particular *g*-sensitive gyro errors while, for the medium to long range trajectories in the test sample (where little wasting was required), only the *sum* had been excited. Thus only the sum was estimable from the test sample, and trying to predict for a trajectory that excited the difference could only lead to great uncertainty. Without the Fisher-based uncertainties that warned the analyst of the situation, the estimate might have been accepted as reasonable.

That this approach to accuracy estimation actually works can be seen from Figure 1. Here estimates of the mean and variance of fundamental level errors have been obtained from a Trident II test program and then propagated to impact on a number of different trajectories to predict what the expected mean and variance of the impact errors should be for that set of flights. The sample mean and variance of the observed impact errors have then been computed to show how well the estimated model was able to predict actual performance. This has been done for two sets of trajectories, a medium range grouping and a short range grouping. The predicted one-sigma ellipses centered at the predicted means are plotted with the heavy lines and the realized sample one-sigma ellipses centered at the realized sample means are plotted with the light lines. Agreement of observed performance with predicted performance is quite good. In particular, note that the estimated model is able to predict the left bias for the medium range grouping and the right bias for the short range grouping.



### Centrifuge Data Processing

Now consider a centrifuge test. The centrifuge will be instrumented to provide very precise measurements of position at various points along its arc. The very same telemetry used in the flight tests provides the corresponding guidance system position which will be differenced with the instrumented measure of position to provide a measure of the guidance system error at multiple times along its "trajectory." Just as for flight tests, these measurements can only be related to the underlying fundamental guidance errors via a model structure as in equation (1). However, because the centrifuge tests are expected to last longer than the guidance portion of the flight tests, and because the centrifuge environment is so different from that of a flight test, an expanded, high fidelity, error model will be necessary. In particular, the longer test times will necessitate adding time-varying guidance errors such as Markovs, random walks, etc. (i.e., driven by process noise). However, the expanded model will still contain the test flight model states as a submodel so that estimates of means and variances of states obtained from test flights can be compared with comparable estimates from centrifuge tests.

The processing of centrifuge data should therefore be virtually identical to the processing of test flight data, the difference being that the position and specific force history that drives the differential equation specifying the model structure in equation (1) is dramatically different. The fact that the specific force, when viewed from the inertially stable platform of the guidance system (on which the gyros and accelerometers are mounted), rotates through a full 360 degrees, repeatedly, greatly improves the observability of the fundamental level errors when compared to a test flight where specific force is in a relatively constant direction. In particular, because the specific force as observed from the stable platform is constantly changing direction, the kind of unobservability cited in the previous section where only the sum of two g-sensitive errors could be observed, but not the difference, would not happen on a centrifuge test.

The data from a sufficient number of independent guidance systems tested on the

centrifuge can be processed (like flight tests) to provide a maximum likelihood estimate of the mean and variance of the fundamental level errors corresponding to those in the flight test model, as well as estimates of the process noise and time constants of any Markovian error states in the expanded centrifuge model (although more complex estimation algorithms are required to provide estimates of process noise and time constants). Associated with these estimates will be the inverse Fisher information matrix to quantify their uncertainty.

### Test For Tactical Representativeness

The centrifuge environment, being so different from the tactical flight environment, leads one to wonder if the centrifuge data is truly representative of what might actually occur in flight. Because of the approach the Navy has taken to cumulative estimation of Trident II accuracy, this question can be quantitatively addressed by comparing estimates from flight data with comparable estimates from centrifuge data.

Let  $\theta_F$  denote the parameters (means and variances) estimated in the flight test model and let  $\theta_S$  denote the supplemental parameters necessary to complete the expanded centrifuge model, so that the centrifuge parameters  $\theta_C$  are  $\theta_C = (\theta_F, \theta_S)$ . Let

$$P_{\theta_C}(Cent) = \begin{bmatrix} P_{\theta_F}(Cent) & P_{\theta_F, \theta_S}(Cent) \\ P_{\theta_S, \theta_F}(Cent) & P_{\theta_S}(Cent) \end{bmatrix}$$

equal the inverse of the Fisher information from the centrifuge estimation and let  $P_{\theta_F}(Flt)$  be the inverse of the Fisher information from the flight test estimation. Similarly, let

$$\hat{\theta}_C(Cent) = (\hat{\theta}_F(Cent), \hat{\theta}_S(Cent))$$

be the estimate from the centrifuge data, and let  $\hat{\theta}_F(Flt)$  be the estimate from the flight data.

Then the centrifuge data can be tested for tactical representativeness by comparing its estimate of  $\theta_F$  with the flight-based estimate of the same parameters. The error in the estimate of  $\theta_F(Cent)$  is asymptotically distributed

$$N(0, P_{\theta_F}(Cent)), \text{ the error in the estimate of}$$

$\theta_F(Flt)$  is distributed  $N(0, P_{\theta_F}(Flt))$ , and the two estimation errors are statistically independent since they arise from completely independent tests. Thus a natural statistical test comparing the two estimates is to form the statistic

$$\chi^2 = [\theta_F(Cent) - \theta_F(Flt)]^T [P_{\theta_F}(Cent) + P_{\theta_F}(Flt)]^{-1} [\theta_F(Cent) - \theta_F(Flt)]$$

which should be distributed Chi-squared on  $p$  degrees of freedom, where  $p$  is the number of parameters in  $\theta_F$ . Thus one can use a Chi-squared table to test for the likelihood of the observed difference in the estimates under the hypothesis that the underlying guidance errors are indeed from the same population. If the test indicates that the observed difference is very unlikely then one should begin to doubt the hypothesis, i.e., one should doubt that centrifuge data is tactically representative.

#### Incorporation of Centrifuge with Flight Data

If the above test indicates that the centrifuge data can be considered tactically representative, then one would like to use it to augment the relatively few flight tests that occur each year. If the error model for the centrifuge data were identical to the error model for the flight data one could simply include the centrifuge data with the flight data in the maximum likelihood processor to get the optimal estimate combining both sources of data. However, the centrifuge error model will be an expanded model and therefore it would not be optimal to process the centrifuge data along with the flight data assuming a flight test model that would not be able to accommodate some of the error mechanisms that are expected in centrifuge data. These additional error mechanisms might get aliased into the parameters of the flight error model that are needed to represent tactical performance. Another possibility would be to use the centrifuge error model to process the combined centrifuge and flight data, in which case, presumably, all the errors would be properly modeled. The difficulty with this approach is that the majority of flight data has already been processed with the flight test error model, and it is an expensive prospect to go back and reprocess all the old flight data with a new error model.

However, as in the previous section, the centrifuge estimate has as a subset, parameters that are common to the flight test model, the parameters  $\theta_F$ , and, associated with this, the variance of the estimation error given by  $P_{\theta_F}(Cent)$ . Similarly the flight-based estimate of  $\theta_F$  has associated with it the variance of its estimation error  $P_{\theta_F}(Flt)$ .

Since these estimation errors are statistically independent, an optimal estimate of  $\theta_F$  can be formed by the information-weighted average of the two estimates:

$$\hat{\theta}_F(Opt) = [P_{\theta_F}^{-1}(Cent) + P_{\theta_F}^{-1}(Flt)]^{-1} \times \{ [P_{\theta_F}(Cent)]^{-1} \hat{\theta}_F(Cent) + [P_{\theta_F}(Flt)]^{-1} \hat{\theta}_F(Flt) \}$$

and the new variance of the estimation error by

$$P_{\theta_F}(Opt) = [P_{\theta_F}^{-1}(Cent) + P_{\theta_F}^{-1}(Flt)]^{-1}.$$

#### Conclusion

The Navy's decision to instrument Trident II missiles and process the resulting test flight data in such a way as to estimate the mean and variance of fundamental level errors may make it possible to augment flight data with centrifuge data, if the centrifuge data is tactically representative. To do so requires processing the centrifuge data in the same manner as for flight test data to obtain a centrifuge-based estimate for comparison with the comparable flight-based estimate.

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# Incorporation of Non-Destructive Centrifuge Tests Into Missile Guidance Assessment

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Dean A. Payne  
Johns Hopkins University Applied Physics Laboratory

Harvey Feltquate  
Charles Stark Draper Laboratory

Title page

## Purpose

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- ◆ As adjunct of Navy's surveillance program for guidance assessment, FBM Guidance Branch is considering a test program to "fly" complete Trident II guidance systems on a centrifuge.
- ◆ Is centrifuge test data tactically representative for accuracy?
- ◆ If so, how to combine with flight data?

10/11/96

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Estimates of the reliability and accuracy of the Trident II guidance system have always been based on flight test data, to the exclusion of factory or lab tests, arguing that flight tests are tactically representative as far as guidance is concerned, while factory and lab tests may not be. However, as the number of flight tests flown each year has diminished, the Navy Fleet Ballistic Missile Guidance Branch is considering modifications to the surveillance program in which additional non-destructive testing would be used to improve early detection of degradation in performance, reliability, and functionality. For the assessment of performance, a prime candidate is a test program in which operational Trident II guidance systems and associated electronics assemblies would be removed from the fleet, "flown" on a centrifuge, and returned to the fleet. This presentation addresses centrifuge testing only as it applies to the assessment of the accuracy of the Trident II guidance system. It does not address the reliability or functionality aspects of centrifuge testing.

Two natural questions arise. First, "How representative of tactical conditions are the centrifuge test results?", and second, "If centrifuge testing is tactically representative, how can it be incorporated with the small number of flight tests to provide an optimal estimate of guidance performance?"

## Centrifuge Vs. Flight Data

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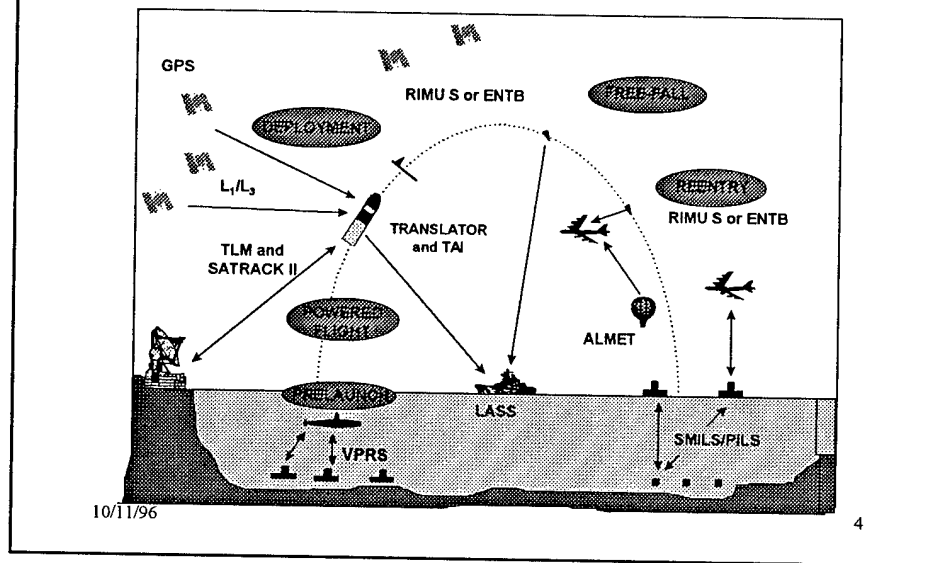
- ◆ Typical assessment of accuracy from flight data via “shoot and score” (sample mean and variance of impact miss).
- ◆ Centrifuge provides no miss data.
- ◆ How can such very different data be quantitatively compared or combined?

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The typical assessment of accuracy from flight test data is via “shoot and score”, i.e., the sample mean and variance of impact miss data is computed, from which a CEP (Circle of Equal Probability) can be calculated. If the Navy had taken such an approach to accuracy assessment of the Trident II system, it would be impossible to quantitatively compare centrifuge and flight data since centrifuge testing provides no impact miss data. Fortunately the Navy has taken a much more sophisticated approach to the assessment of accuracy from flight data, and, as will be seen in the remainder of the presentation, it is this approach which allows a quantitative comparison, and, ultimately, combining of the two data sources to form an optimal estimate of accuracy. The key to the approach is to instrument the various subsystems sufficiently to allow the estimation of the fundamental level error sources (accelerometer and gyro biases, etc.) that cause the inaccuracy.

## Flight Test Setup



The Navy's approach to accuracy evaluation is founded on the instrumentation of various phases and subsystems. From the inception of the Trident II flight test program the Navy has planned for this enhanced data gathering and processing so that the instrumentation is an integral part of the tactical system design, and the instrumented data is therefore typical of a tactical system. For the assessment of guidance accuracy, the instrumentation is a GPS-based missile tracking system called SATRACK II that provides precise measurements of the range and range rate to the various satellites in view.

## Flight Data for Guidance Evaluation

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- ◆ Telemetry provides IMU's position and velocity from which compute range and range rate to satellites.
- ◆ SATRACK tracking system provides measured range and range rate.
- ◆ Difference provides measured IMU error (equivalent to position and velocity error).

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Telemetry provides the guidance system (or IMU - Inertial Measurement Unit) computed position and velocity, from which the range and range rate to the various satellites can be calculated. Differencing with the SATRACK provided measure of range and range rate provides a measure of the error in the IMU's computation of position and velocity.

But multiple measurements of errors in the IMU's position and velocity do not readily translate into estimates of accelerometer and gyro biases, etc.. How does the SATRACK data enable one to estimate these fundamental level errors?

## Accuracy Evaluation: Flights

- ◆ Approach relies on a mathematical error model *structure* relating fundamental level errors (accelerometer and gyro biases, etc.) to measured errors (position and velocity).
  - Takes form of 1st order matrix differential equation obtained by 1st order perturbation of navigation equations and basic physics.

$$\begin{aligned} \dot{x} &= Fx + Gy, \quad x(0) = x_0 \text{ (IC errors)} \\ y &= 0, \quad y(0) = y_0 \text{ (Guidance errors)} \end{aligned}$$

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The approach relies on a mathematical error model structure that relates the fundamental level errors to the measured errors. This error model structure is obtained by a first order perturbation of the navigation equations used by the guidance system plus basic physics. For example the navigation equations integrate specific force plus computed gravity to compute velocity, which is, in turn, integrated into position. An accelerometer bias will therefore lead to an error in specific force which will integrate into an error in computed velocity and, after a second integration, into an error in computed position. Similar derivations can be made for other accelerometer and gyro errors, leading to a first order matrix differential equation that describes how the fundamental level errors integrate into errors in position and velocity at any timepoint along the trajectory.

In this equation the guidance errors  $y$  are modeled as being random constants (the derivative of  $y$  is zero) because the Mk-6 guidance system inertial components are very stable and the boost phase time of flight is short. The vector  $x$  denotes the 9-vector of the position, velocity, and platform orientation errors and is not constant, but changes throughout the flight depending on the trajectory and guidance errors that are active. The solution to the differential equation depends on the initial values of  $x$  and  $y$  as well as on the trajectory, through the position and specific force history that are implicit in the functions  $F$  and  $G$ .

## Accuracy Evaluation: Flights

- ◆ Solution a function of position and specific force along the trajectory through F and G
  - Provides sensitivity of errors in position and velocity to IC and guidance errors at any point along trajectory (e.g., impact).
- ◆ Stacking sensitivity matrices at SATRACK measurement times leads to data equation

$$Z = H \begin{bmatrix} x \\ y \end{bmatrix} + v, v \sim N(0, R)$$

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Because the solution depends on the initial value, the partial derivative of the solution with respect to the initial values (for the IC and guidance errors) can be computed. These partial derivatives provide the sensitivity of the position, velocity, and orientation errors to the IC and guidance errors at any point along the trajectory. Because the solution is itself dependent on the trajectory (through the functions F and G) the sensitivity coefficients will also be trajectory dependent. For example an accelerometer bias will have a much greater impact effect on a trajectory with a long time of flight than on one with a short time of flight.

If the sensitivity matrices are computed at every SATRACK measurement time, the effect of the IC and guidance errors on each measurement can be computed. Stacking the measurements and the associated sensitivity matrices leads to the data equation, where Z is the stacked measurements, H the stacked sensitivity matrices, and v the measurement noise assumed to be normally distributed (Gaussian) with zero mean and variance R. Given a sufficient number of measurements and sufficient variation in the sensitivity coefficients, the system of linear equations can be solved for the IC and guidance errors.

## Accuracy Evaluation: Flights

- ◆ System of linear equations provides ML estimate of IC and guidance errors realized on particular flight.
- ◆ Stacking data equations for all tests in sample allows ML estimation of mean and variance of IC and guidance errors that characterize errors across all tests.
  - Inverse Fisher information matrix  $P$  provides quantified uncertainty in estimate

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The data equation can be solved for the maximum likelihood (ML) estimate of the IC and guidance errors that were realized on the particular flight. The uncertainty in the estimate is provided by the inverse of the so-called Fisher information matrix which captures the “observability” of the errors. For example, a scale factor error on the y-accelerometer is unobservable if the trajectory is such that no thrust (specific force) occurs along the y-axis.

By stacking all the data equations across each of the flight tests in the test program one can in a similar manner estimate, not a particular realization of an error on a given flight, but rather the mean and variance that characterize the errors across all the flights in the test program. A maximum likelihood estimator is used and, once again, associated with the estimate is the Fisher information matrix whose inverse provides the uncertainty in the estimate (more precisely, the variance of the estimation error). Given a sufficient number of flights, the error in the maximum likelihood estimate of the mean and variance is distributed  $N(0,P)$  where  $P$  is the inverse of the Fisher information. Thus, not only can the mean and variance of fundamental level guidance errors be estimated, but the quality of the estimate can also be provided. An estimate without some indication of its quality is not worth much.



## Accuracy Evaluation: Flights

- ◆ Accuracy (CEP) for any given trajectory can be obtained by using the impact sensitivity matrix to propagate estimated mean and variance of fundamental errors to impact.
  - Propagated inverse Fisher matrix yields uncertainty in accuracy estimate.

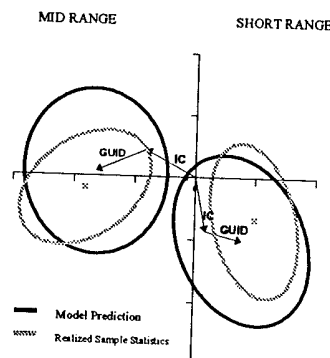
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The object of the test flights is to provide an estimate of the accuracy of the system. But estimates of the mean and variance of fundamental level errors do not provide an estimate of accuracy. Just as the error model structure was necessary to estimate fundamental level errors, it is also necessary to translate estimates of the mean and variance of fundamental level errors into a prediction of accuracy. The model structure allows the computation of sensitivity coefficients at any point along the trajectory, in particular, at impact. The impact sensitivity matrices can then be applied to the mean and variance of fundamental level errors to generate the predicted mean and variance at impact from which a CEP can be calculated. The sensitivity matrices can also be used to propagate the inverse Fisher matrices to impact to provide a quantified uncertainty in the accuracy estimate. Note that this prediction of accuracy can be done even for untested trajectories provided a three-degree-of-freedom missile simulator is available to generate the missile position and specific force history.

It should be noted that when propagated to impact all the information gathered from the SATRACK data over all the trajectories in the test program is collapsed to only five parameters, the mean and variance and covariance, of downrange and crossrange errors, so that the asymptotic normality of the estimation errors applies at impact for relatively few flights -- as few as a dozen in the Trident II experience.

## Example: Predicted Vs. Realized Sample Statistics



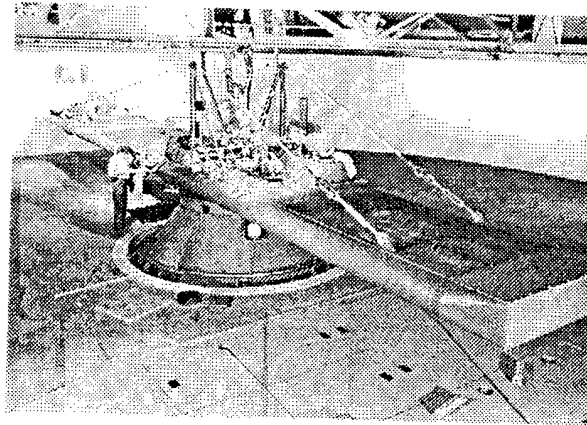
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To show that this approach to accuracy evaluation really works consider this figure. Here data from a number of missile flights has been processed to estimate the mean and variance of the fundamental level guidance and IC errors. These estimates have then been propagated to impact on a number of mid range and short range trajectories via the sensitivity matrices for each. From the resulting impact means and variances (of downrange and crossrange miss) one can predict the sample mean and variance for the flights in the mid range grouping, and also for the flights in the short range grouping. (The uncertainty in the prediction can also be computed.) One can then compute the actual sample mean and variance of the realized impact misses and compare to the prediction. The figure indicates that the agreement between realized and predicted sample statistics is quite good. In particular the data-derived model was able to predict the left bias for the mid range grouping and the right bias for the short range grouping.

Now let's consider the centrifuge to see how it is that this approach to accuracy evaluation is able to allow the comparison and ultimate combining of flight and centrifuge data.

## Centrifuge

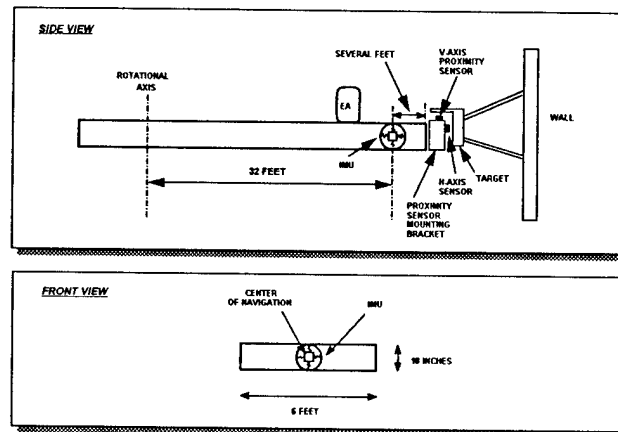


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The centrifuge has an arm 35 feet in radius, but the IMU is mounted 32 feet from the center. Thus, since centripetal acceleration is given by radius times angular rate (in Radians) squared, 1 radian/sec yields one g of acceleration, 2 radians/sec yields 4 g's, and pi radians/sec yields about 10 g's. Since one full circle is 2 pi radians, this centrifuge is capable of generating 10 g's of acceleration with only one revolution every two seconds. This picture is of the centrifuge as it was this spring. It is now being refurbished and instrumentation is being mounted on the walls to precisely measure position.

# Centrifuge Test Setup



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This figure shows a schematic of the brackets mounted on the wall supporting the proximity sensors that are able to measure position to within 20 thousandths of an inch

## Centrifuge Data

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- ◆ Telemetry provides IMU's computed position.
- ◆ Instrumentation along wall provides measured position.
- ◆ Difference provides measure of IMU position error at multiple points along IMU's arc.

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Just as SATRACK provides the position and velocity errors for the IMU in flight, the instrumentation along the wall provides precise measurement of the IMU's position which can be differenced with the IMU's computed value to provide a measure of its error. This is done at multiple points along the arc of the IMU just as SATRACK measurements are made at multiple points along the trajectory of the IMU.

## Accuracy Evaluation: Centrifuge

- ◆ Apply exactly the same process to centrifuge data as for flight data.
  - Estimate fundamental guidance errors and obtain inverse Fisher to quantify uncertainty.
- ◆ Difference lies in radically different “trajectory” driving differential equation.
  - Rotating specific force vector provides better observability than flight test where specific force changes direction only slowly.

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The exact same process that was applied to flight data can now be applied to centrifuge data to estimate the fundamental level guidance errors from the sequence of measured position errors as the centrifuge increases and then decreases speed, in other words, as the g's rise and fall. Once again the inverse of the Fisher information matrix associated with the maximum likelihood estimation process provides the uncertainty in the estimate. Recall that the ability to estimate fundamental level errors is a result of the error model structure defined by the differential equation given earlier which is driven by the position and specific force history of the IMU's trajectory. To the estimation process, the only difference between a centrifuge test and a flight test lies in the radically different “trajectory” that drives the error structure differential equation. For a centrifuge the IMU's position follows a 32 foot circle and the specific force appears to rotate relative to the inertially stable platform at about 180 degrees/second at 10 g's. Contrast this with a test flight where the position follows a more or less elliptical path downrange and the specific force changes direction only slowly, at least until after third stage burnout when the magnitude of the specific force is negligible.

The fact that the specific force is constantly changing direction significantly improves the observability of the fundamental level errors compared to a flight test. It allows much better discrimination of the acceleration sensitive accelerometer and gyro errors from one another.

## Comparison of Centrifuge And Flight Estimates

- ◆ Is centrifuge data tactically representative?
  - Want to compare with flight data since it is tactically representative.
- ◆ Now have common ground on which to perform statistical test.
  - Chi-square test on independent estimates:

$$\chi = (\hat{\theta} - \hat{\theta}_c)' [P_c + P_f] (\hat{\theta} - \hat{\theta}_c)$$

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Although the unusual “trajectory” followed by an IMU on a centrifuge allows for better estimation of fundamental level errors, it also leads one to wonder if the errors realized on a centrifuge test are really typical of what would be realized on a flight. Given a sufficient number of centrifuge tests we might be able to estimate the mean and variance of the fundamental level errors realized in the centrifuge test program very well, but the estimates may not be typical of what would be realized tactically. Since flight test data is tactically representative for guidance, we can test the centrifuge estimates for tactical representativeness by comparing with the comparable estimates from flight data, and we now have a common ground on which to make the comparison, since both test programs provide estimates of the mean and variance of fundamental level errors,  $\theta$  sub F for flight and  $\theta$  sub C for centrifuge, together with a quantified uncertainty in that estimate, what I’ve denoted  $P$  sub F for flight and  $P$  sub C for centrifuge. Then, because the estimation errors asymptotically have a normal distribution with zero mean and variance  $P$  sub F or C, one can test to see if the flight and centrifuge estimates characterize the same distribution by comparing the indicated statistic  $\chi$  against a Chi-squared table where the degrees of freedom is the number of parameters estimated. If the realized value of  $\chi$  is very unlikely then it is unlikely that centrifuge data is typical of flight data. By propagating both estimates and inverse Fishers to impact on a common trajectory, this test can also be applied where the asymptotic normality of the estimation error is more likely to be valid and where the real interest in the estimates lies -- namely at impact.

## Optimal Combining of Estimates

- ◆ If test for tactical representativeness is passed, would like to combine centrifuge and flight data to form best estimate.
- ◆ Information-weighted average (via Fisher information from each data source) of estimated mean and variance of fundamental level errors.

$$\theta_{\text{opt}} = [P_{\text{c}}^{-1} + P_{\text{f}}^{-1}]^{-1} [P_{\text{c}}^{-1} \theta_{\text{c}} + P_{\text{f}}^{-1} \theta_{\text{f}}]$$

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If the test for tactical representativeness is passed, either at the fundamental level or at impact for a representative class of trajectories, one would then like to utilize the centrifuge data to improve the estimate of accuracy obtained from the limited flight test program. Because the estimation process provides a quantified uncertainty in each estimate, namely the inverse of the Fisher information, one can optimally combine the two estimates by computing the information-weighted average as shown. The uncertainty in the combined estimate is given by the first term in brackets on the right side of the equality sign, that is, by the inverse of the sum of the information from each test program. Once again, this combining can be done at the fundamental level, or at impact after propagating both estimates (and inverse Fishers) to impact on a common trajectory. Of course, once one has an optimal estimate of the mean and variance at impact for any trajectory, the CEP and its uncertainty can also be computed via standard formulas.



## Conclusion

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- ◆ Navy's decision to instrument Trident II missiles, and process resulting data so as to estimate mean and variance of fundamental errors, makes possible comparison and combining of centrifuge and flight data.
- ◆ Contrast with situation if Navy had only resorted to "shoot and score" for Trident II performance evaluation.

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In conclusion I'd like to emphasize that the testing of guidance centrifuge data for tactical representativeness and its combining with flight data to form an optimal estimate would simply be impossible if it were not for the approach the Navy has taken to the estimation of accuracy from the inception of the flight test program. The very approach taken for the flight test program can be taken over in total to the centrifuge test program, and by so doing provides a common ground on which the estimates from the two test programs can be compared and, if deemed comparable, combined to form an optimal estimate of guidance accuracy, pooling both sets of information. If the Navy had taken the usual "shoot and score" approach to accuracy evaluation from its flight test program, there would be no quantifiable way to compare or combine the results of the two test programs.

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